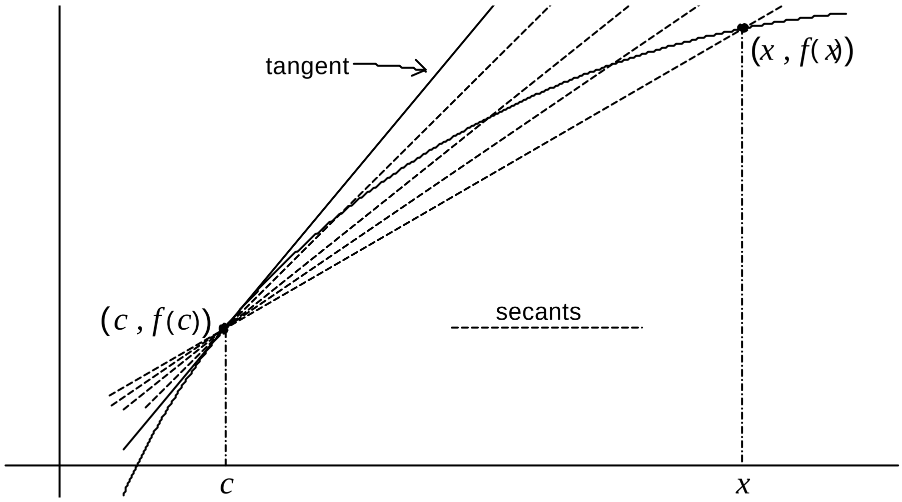
THE GRADIENT OF THE TANGENT AS THE LIMIT
OF THE GRADIENT OF THE SECANT

Consider what happens as the secant approaches a single point on the curve.



The gradient of the secant *approaches* the gradient of the tangent. We say that the gradient of the tangent is the *limit* of the gradient of the sequence:

gradient of the tangent at the point on the curve where $x = c$

$= \lim_{x \to c}$ (gradient of secant)

$$= \lim_{x \to c} \frac{\Delta f}{\Delta x}$$

$$= \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

The gradient of the tangent to a curve can be estimated numerically by calculating the gradients of secants. Consider the point $(3,5)$ on the graph of $f(x) = x^3 - 5x - 7$.

x	$f(x)$	gradient of secant $= \frac{\Delta f}{\Delta x} = \frac{f(x) - f(3)}{x - 3}$
4	37.000000	32.000000
3.5	18.375000	26.750000
3.1	7.291000	22.910000
3.01	5.220901	22.090100
3.001	5.022009	22.009001
3.0001	5.002200	22.000900

From the table, the gradient of the secant seems to be approaching 22. It seems reasonable to conclude that:

gradient of tangent at point on curve where $x = 3$

$$= \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$$

$= 22$