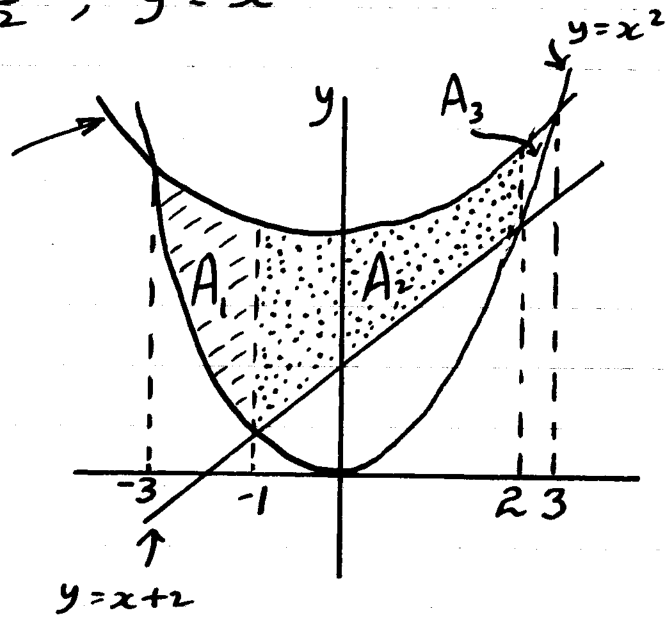
Find the area bounded by the graphs

$y = x+2$, $y = \frac{1}{2}x^2 + \frac{9}{2}$, $y = x^2$



Values of x at points of intersection found using TI83 intersection features.

Using integration feature on TI83:

$$A_1 = \int_{-3}^{-1} \left(\tfrac{1}{2}x^2 + \tfrac{9}{2}\right) - x^2 \, dx = 4\tfrac{2}{3} \text{ sq units}$$

$$A_2 = \int_{-1}^{2} \left(\tfrac{1}{2}x^2 + \tfrac{9}{2}\right) - (x+2) \, dx = 7\tfrac{1}{2} \text{ sq units}$$

$$A_3 = \int_{2}^{3} \left(\tfrac{1}{2}x^2 + \tfrac{9}{2}\right) - x^2 \, dx = 1\tfrac{1}{3} \text{ sq units}$$

total area

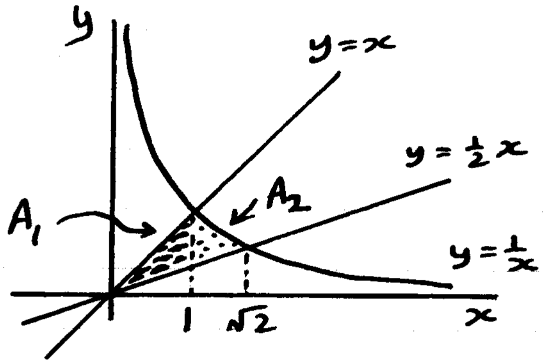
$= 4\tfrac{2}{3} + 7\tfrac{1}{2} + 1\tfrac{1}{3}$

$= 13\tfrac{1}{2}$ sq units

Find the area bound by the graphs

$y = x$, $y = \frac{1}{2}x$ and $y = \frac{1}{x}$

Values of x at points of intersection found using TI 83 intersection feature.



$$A_1 = \int_0^1 x - \frac{1}{2}x \; dx$$

$$A_2 = \int_1^{\sqrt{2}} \frac{1}{x} - \frac{1}{2}x \; dx$$

by integration:

A_1

$= \int_0^1 \frac{1}{2}x \; dx$

$= \left[\frac{1}{4}x^2 \right]_0^1$

$= \frac{1}{4} \times 1^2 - \frac{1}{4} \times 0^2$

$= \frac{1}{4}$ sq units

A_2

$= \int_1^{\sqrt{2}} \frac{1}{x} - \frac{1}{2}x \; dx$

$= \left[\ln|x| - \frac{1}{4}x^2 \right]_1^{\sqrt{2}}$

$= \left(\ln\sqrt{2} - \frac{1}{4}(\sqrt{2})^2\right)$
$\quad - \left(\ln 1 - \frac{1}{4} \times 1^2\right)$

$= \ln\sqrt{2} - \frac{1}{2} - 0 + \frac{1}{4}$

$= \ln\sqrt{2} - \frac{1}{4}$ sq. units

total area

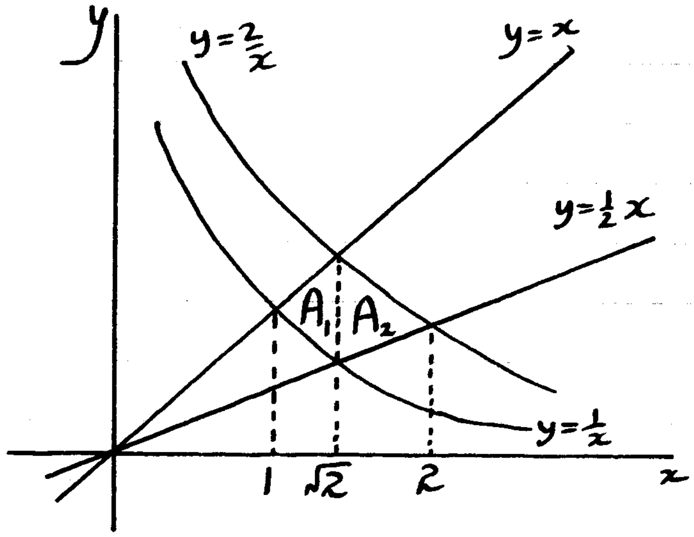
$= \frac{1}{4} + \ln\sqrt{2} - \frac{1}{4}$

$= \ln\sqrt{2}$

$= \frac{1}{2}\ln 2$ sq units

Find the area bounded by the graphs
$y = x,\ y = \frac{1}{2}x,\ y = \frac{1}{x},\ y = \frac{2}{x}$

Values of x at points of intersection found using TI83 intersection feature.



A_1

$= \int_1^{\sqrt{2}} x - \frac{1}{x}\, dx$

$= \left[\frac{1}{2}x^2 - \ln|x|\right]_1^{\sqrt{2}}$

$= \left(\frac{1}{2}\times 2 - \ln\sqrt{2}\right) - \left(\frac{1}{2}\times 1^2 - \ln 1\right)$

$= (1 - \ln\sqrt{2}) - (\frac{1}{2} - 0)$

$= \frac{1}{2} - \ln\sqrt{2}$

$= \frac{1}{2} - \ln 2^{1/2}$

$= \frac{1}{2} - \frac{1}{2}\ln 2$ sq units

A_2

$= \int_{\sqrt{2}}^{2} \frac{2}{x} - \frac{1}{2}x\, dx$

$= \left[2\ln|x| - \frac{1}{4}x^2\right]_{\sqrt{2}}^{2}$

$= \left(2\ln 2 - \frac{1}{4}\times 2^2\right) - \left(2\ln\sqrt{2} - \frac{1}{4}\times 2\right)$

$= 2\ln 2 - 1 - 2\ln\sqrt{2} + \frac{1}{2}$

$= 2\ln 2 - 2\ln 2^{1/2} - 1/2$

$= 2\ln 2 - 2\times\frac{1}{2}\ln 2 - 1/2$

$= 2\ln 2 - \ln 2 - \frac{1}{2}$

$= \ln 2 - \frac{1}{2}$ sq units.

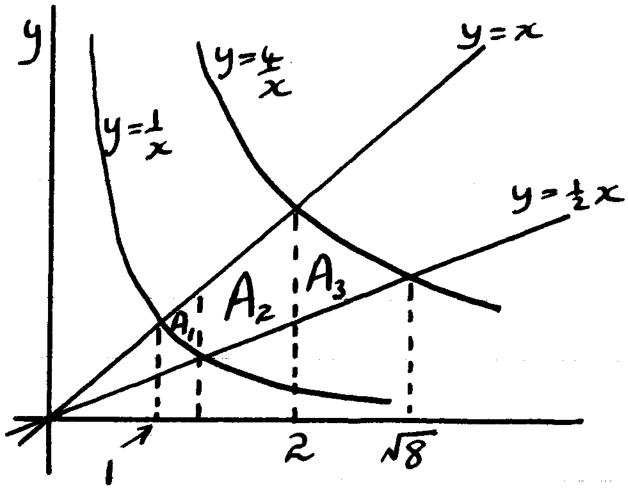
$\therefore$ total area

$= \frac{1}{2} - \frac{1}{2}\ln 2 + \ln 2 - \frac{1}{2}$

$= \frac{3}{2}\ln 2$ sq units

Find the area bounded by the graphs

$y = x$, $y = \frac{1}{2}x$, $y = \frac{1}{x}$, $y = \frac{4}{x}$



Values of x at points of intersection found using TI83 intersection feature.

$A_1 = \frac{1}{2} - \frac{1}{2}\ln 2$ sq units (see previous problem)

A_2

$= \int_{\sqrt{2}}^{2} x - \frac{1}{2}x \, dx$

$= \int_{\sqrt{2}}^{2} \frac{1}{2}x \, dx$

$= \left[\frac{1}{4}x^2\right]_{\sqrt{2}}^{2}$

$= \frac{1}{4}\times 2^2 - \frac{1}{4}\times(\sqrt{2})^2$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$ sq unit

A_3

$= \int_{2}^{\sqrt{8}} \frac{4}{x} - \frac{1}{2}x \, dx$

$= \left[4\ln|x| - \frac{1}{4}x^2\right]_2^{\sqrt{8}}$

$= \left(4\ln\sqrt{8} - \frac{1}{4}\times(\sqrt{8})^2\right) - \left(4\ln 2 - \frac{1}{4}\times 2^2\right)$

$= 4\ln\sqrt{8} - 2 - 4\ln 2 + 1$

$= 4\ln 8^{\frac{1}{2}} - 4\ln 2 - 1$

$= 4\times\frac{1}{2}\ln 8 - 4\ln 2 - 1$

$= 2\ln 2^3 - 4\ln 2 - 1$

$= 2\times 3\ln 2 - 4\ln 2 - 1$

$= 6\ln 2 - 4\ln 2 - 1$

$= 2\ln 2 - 1$ sq units

$\therefore$ total area

$= \frac{1}{2} - \frac{1}{2}\ln 2 + \frac{1}{2} + 2\ln 2 - 1$

$= \frac{3}{2}\ln 2$ sq units