BINOMIAL PROBABILITY DISTRIBUTION

First consider a Bernouilli trial.

A Bernouilli trial has only two outcomes - success or failure.

Examples are:

Bernouilli trial	success	probability of success p	failure	probability of failure q
flip of a coin	heads	$\frac{1}{2}$	not heads	$\frac{1}{2}$
draw card from pack	heart	$\frac{1}{4}$	not a heart	$\frac{3}{4}$
throw a six-faced die	multiple of 3	$\frac{1}{3}$	not a multiple of 3	$\frac{2}{3}$

For a Bernouilli trial, $p+q=1$

The binomial probability distribution occurs in the following circumstances:

> There is a fixed number of Bernouilli trials (n).
> For each Bernouilli trial, the probability of success (p) is constant.
> We are interested in the number of successes.

BINOMIAL PROBABILITIES USING A TREE DIAGRAM SHOWING EQUALLY LIKELY OUTCOMES

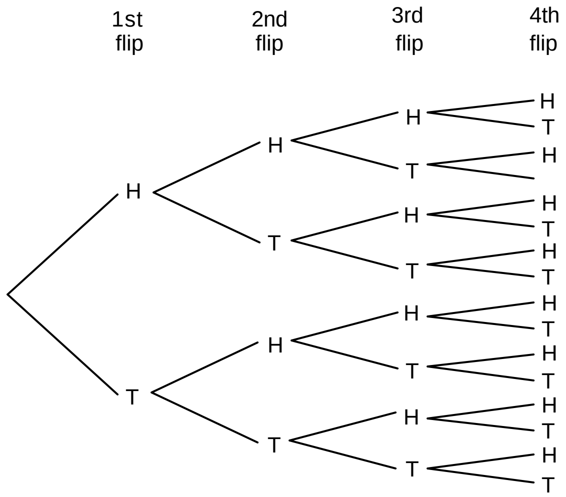
Consider a fair coin which is flipped 4 times. Let Y be the number of heads. This is an example of the binomial probability distribution:

> There are 4 Bernouilli trials ($n=4$).
> The probability of success is constant ($p=\frac{1}{2}$).
> Y is the number of successes.

The probabilities can be calculated from a tree diagram showing 16 equally likely outcomes.

The probability distribution of Y is:

value of Y	probability
0	$\frac{1}{16}=0\cdot 0625$
1	$\frac{4}{16}=0\cdot 25$
2	$\frac{6}{16}=0\cdot 375$
3	$\frac{4}{16}=0\cdot 25$
4	$\frac{1}{16}=0\cdot 0625$
	1



BINOMIAL PROBABILITIES FROM TABLES

Binomial probability distributions can be obtained from a binomial probability table. The distribution for Y is obtained by looking up:

probability of success $p = \frac{1}{2}$
number of trials $n = 4$
number of successes $r = 0, 1, 2, 3, 4$

	r	p 0·50	
$n = 4$	0	0·0625	
	1	0·2500	
	2	0·3750	
	3	0·2500	
	4	0·0625	

Use left hand column for r with top row for p.
Use right hand column for r with bottom row for p.

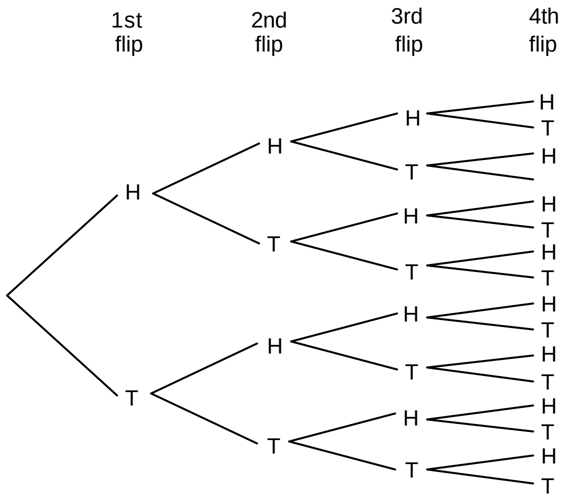
BINOMIAL PROBABILITIES USING PASCAL'S TRIANGLE

Consider a coin which is flipped 4 times. Suppose the coin is not fair and that heads is expected to occur 3 times for every 2 times that tails occurs ie. heads is 50% more likely than tails. Let Y be the number of heads. This is an example of the binomial probability distribution:

> There are 4 Bernouilli trials ($n = 4$).
> The probability of success is constant ($p = \frac{3}{5}$).
> Y is the number of successes.

The tree diagram now shows outcomes which are not equally likely. Label each branch with a probability ($\frac{3}{5}$ or $\frac{2}{5}$). Multiply the probabilities. Use the results to show that the probability distribution of Y is:

value of Y	probability
0	$\left(\frac{2}{5}\right)^4 = 0 \cdot 0256$
1	$4\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^3 = 0 \cdot 1536$
2	$6\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2 = 0 \cdot 3456$
3	$4\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right) = 0 \cdot 3456$
4	$\left(\frac{3}{5}\right)^4 = 0 \cdot 1296$
	1



The pattern in the powers of $\frac{3}{5}$ and $\frac{2}{5}$ relates to the number of successes (heads) and failures (tails). The numbers 1, 4, 6, 4, 1 represent the numbers of different arrangements of successes and failures .

value of Y	arrangements
0	TTTT
1	HTTT THTT TTHT TTTH
2	HHTT HTHT HTTH THHT THTH TTHH
3	THHH HTHH HHTH HHHT
4	HHHH

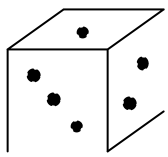
$\text{P}(x \text{ successes from } n \text{ trials}) = \text{ number of arrangements } \times p^x \times q^{n-x}$

The number of arrangements can be found in Pascal's Triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
etc.

Each number in Pascal's Triangle is the sum of the pair of numbers immediately above.

<u>EXAMPLE OF A BINOMIAL PROBABILITY USING PASCAL'S TRIANGLE</u>



Consider 5 throws of a six-faced die. What is the probability of throwing 2 sixes?

Let X be the number of sixes. This is an example of the binomial probability distribution:

> There are 5 Bernouilli trials ($n = 5$).
> The probability of success is constant ($p = \frac{1}{6}$).
> X is the number of successes.

Choose the number of arrangements from Pascal's Triangle. Because there are 5 Bernouilli trials, the row starts 1, 5, ...

number of arrangements		1	5	**10**	10	5	1
				↑			
number of successes	→	0	1	2	3	4	5

$\text{P}(2 \text{ sixes from } 5 \text{ throws})$
$= \text{P}(\text{X} = 2)$
$= \text{number of arrangements} \times p^x \times q^{n-x}$
$= 10 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$
$\approx 0 \cdot 1608$

BINOMIAL PROBABILITIES USING COMBINATIONS

Instead of using Pascal's Triangle to find the number of arrangements, combinations can be used.

For example, the number of arrangements of 2 successes in 5 Bernouilli trials is the combination:

$^5C_2$ which can also be written $\binom{5}{2}$

(N.B. This concept is covered in greater depth in Mathematics C.)

The calculator provides the value $^5C_2 = 10$.

Therefore, in the previous example of throwing a six-faced die:

$$
\begin{aligned}
&\mathrm{P}(\text{2 sixes from 5 throws}) \\
&= \mathrm{P}(\mathrm{X} = 2) \\
&= {}^nC_x \times p^x \times q^{n-x} \\
&= {}^5C_2 \left(\tfrac{1}{6}\right)^2 \left(\tfrac{5}{6}\right)^3 \\
&= 10 \left(\tfrac{1}{6}\right)^2 \left(\tfrac{5}{6}\right)^3 \\
&\approx 0 \cdot 1608
\end{aligned}
$$