MODELLING WITH QUADRATIC FUNCTIONS

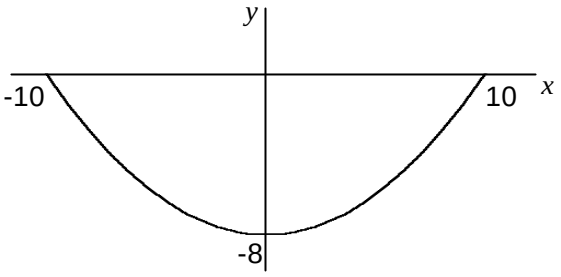
Example of modelling by a quadratic function - Consider a rope suspended from two points 20 metres apart so that the lowest point of the rope is 8 metres below those points. Assume that the position of the rope can be represented by a quadratic function. Place the axes as shown in the diagram. The turning point is $(0,-8)$ and the equation is in the form:

Substitute $h=0$ and $k=^{-}8$ in the completed square form of the equation for a quadratic function:

$$y=a(x-h)^2+k$$

$$y=a(x-0)^2-8$$

$$y=a\,x^2-8$$



when $x=10,\ y=0$:

$$0=a\times 10^{\ 2}-8$$

$$a=\tfrac{8}{100}$$

$$a=\tfrac{2}{25}$$

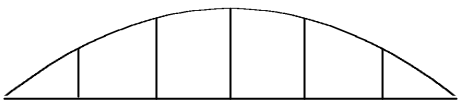
Therefore the equation of the curve is $y=\frac{2}{25}x^2-8$

PROBLEMS

1) Consider the number of handshakes which occur in a room when each person shakes hands with every other person. Find the number of handshakes as a function of the number of people in the room.

people	0	1	2	3	4	5	6	7	...
handshakes	0	0	1	3	?	?	?	?	...

2) The span of a suspension bridge is 30 metres long and the length of the central cable is 10 metres. Calculate the lengths of the other supporting cables.

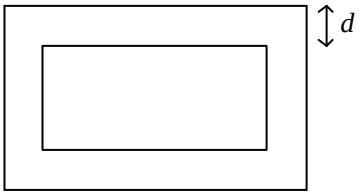


3) A farmer uses 1000 metres of fencing to make a rectangular paddock. If a creek forms one side of the paddock, what are the dimensions of the paddock if the area is to be as large as possible?

4) In an experiment, a rock is projected vertically upwards and its height above its point of projection is measured at one second intervals with the following results. Draw a graph of height against time and find a suitable quadratic function to model the data.

time (s)	0	1	2	3	4	5	6	7	8	9
height (m)	0	46	81	107	120	123	121	103	78	44

5) A 1000 m by 500 m rectangular paddock is ploughed by a farmer starting at the perimeter and moving gradually towards the centre. The tractor travels at 10 km/h and the width of the plough is 10 metres. The farmer notices that the tractor moves towards the centre slowly at first but gradually speeds up. Suppose the farmer ploughs a strip of width d metres in t hours. Write t as a function of d.



6) The braking distance needed to stop a car is proportional to the square of the speed. If a car travelling at 15 m/s can stop in 12·5 m, what is the stopping distance if the speed is 40 m/s?