

MATRICES AND GROUPS

1. Show that the set of matrices $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in R \text{ and } a + b = 1 \right\}$ is not closed under matrix multiplication.

2. Show that the set of matrices $\left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in R \text{ and } a \neq 0 \right\}$ is closed under matrix multiplication. Does the set form a group under matrix multiplication?

3. Complete a Cayley table for the following matrices under matrix multiplication.

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Show that the matrices form a group under matrix multiplication.

4. Complete a Cayley table for the following matrices under matrix multiplication.

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

Show that the matrices form a group under matrix multiplication.