**APPLICATION OF THE MULTIPLICATION AND ADDITION PRINCIPLES**

1. The letters ENGLISH are arranged to make 7 letter words without repetition. How many words can be made if:
   a) there are no restrictions
   b) the first letter is E
   c) E is the first or last letter
   d) the first and last letters are vowels
   e) the first letter is a consonant
   f) the first and last letters are consonants
   g) the E and I occur next to each other in the order EI
   h) the E and I occur next to each other
   i) the E, N and G occur next to each other?

2. How many words of 7 letters or less can be made from the letters ENGLISH if:
   a) repetition is not allowed b) repetition is allowed?

3. The letters FACTORISE are arranged to make 9 letter words. How many words can be made without repetition if:
   a) there are no restrictions
   b) the first and last letters are vowels
   c) the first and last letters are consonants
   d) all the vowels occur at the beginning of the word
   e) the first letter is a vowel and the last letter is a consonant
   f) the vowels occur next to each other
   g) the consonants occur next to each other
   h) the consonants and vowels alternate?

4. How many words of 7 letters or less can be made from the letters FACTORISE if:
   a) repetition is not allowed
   b) repetition is allowed?

5. How many ways can 3 Mathematics books, 4 Science books and 5 History books be arranged on a shelf if:
   a) there are no restrictions
   b) the Mathematics books are together
   c) the books for each subject are together
   d) the book at each end of the row is a Science book
   e) the book at the left hand end is a Mathematics book and the book at the right hand end is a History book?

6. Three married couples (the Smiths, Jones and Browns) sit on a bench. In how many ways can this be done if:
   a) there are no restrictions
   b) the Smiths sit at the ends
   c) the Smiths sit together
   d) each married couple sits together?

7. Three married couples (the Smiths, Jones and Browns) sit at a circular table. In how many ways can this be done if:
   a) there are no restrictions
   b) the Smiths sit together
   c) each married couple sits together

8. How many ways can 2 men, 3 women and 4 children sit in a row if:
   a) there are no restrictions
   b) all the children sit together
   c) both men sit together, all the women sit together and all the children sit together

9. How many ways can 2 men, 3 women and 4 children sit in a circle if:
   a) there are no restrictions
   b) all the children sit together
   c) both men, all the women and all the children sit together

10.
   a) How many 4 digit integers are odd?
   b) How many 4 digit integers are multiples of 5?

11. How many 4 digit integers can be made from the digits 1, 3, 4, 5, 6, 7 if:
   a) repetition is not allowed  b) repetition is allowed?

12. How many even 4 digit integers can be made from the digits 1, 3, 4, 5, 6, 7 if:
   a) repetition is not allowed  b) repetition is allowed?

13. How many 4 digit integers can be made from the digits 0, 1, 3, 4, 5, 6, 7 if:
   a) repetition is not allowed  b) repetition is allowed?

14. How many even 4 digit integers can be made from the digits 0, 1, 3, 4, 5, 6, 7 if:
   a) repetition is not allowed  b) repetition is allowed?

15. How many integers of 3 digits or less can be formed with the digits 2, 4, 5, 6, 7, 8 if:
   a) repetition is not allowed  b) repetition is allowed?

16. How many integers of 3 digits or less can be formed with the digits 0, 2, 4, 5, 6, 7, 8 if:
   a) repetition is not allowed  b) repetition is allowed?

17. How many even integers of 3 digits or less can be formed with the digits 2, 4, 5, 6, 7, 8 if:
   a) repetition is not allowed  b) repetition is allowed?

18. How many even integers of 3 digits or less can be formed with the digits 0, 2, 4, 5, 6, 7, 8 if:
a) repetition is not allowed  b) repetition is allowed?

19. How many different 3 letter words can be made using letters from the word TABLE if repetition is not allowed?

20. How many different 5 letter words can be made using letters from the word MATHEMATICS if repetition is not allowed?

21. How many different words can be made using letters from the word MARKOV if repetition is not allowed?

22. How many different words of 5 letters or less can be made using all the of the alphabet if repetition is not allowed?

**APPLICATION OF COMBINATIONS**

GOLD LOTTO

- A game consists of choosing 6 integers from 1 to 45.
- Eight numbers are drawn one at a time at random.
- The first six numbers drawn are the main numbers and the last two numbers drawn are the supplementary numbers.
- To win the first division prize, you must have a game which is the same as the six main numbers.

1. How many different games are possible?

2. How many games are possible if a person chooses:
   a) even numbers
   b) prime numbers
   c) multiples of 5
   d) numbers less than 20
   e) numbers which include the digit 2?

3. How many games are possible for Susan if she chooses:
   a) her age and five other numbers
   b) numbers which do not include her age
   c) the ages of her family (mum, dad, sister and herself) and two other numbers
   d) numbers which do not include the ages of her family (mum, dad, sister and herself)?

4. How many games are possible for David if:
   a) his favourite numbers 21 and 31 are included
   b) one but not both of his favourite numbers 21 and 31 are included
   c) his unlucky numbers 13, 26 and 39 are excluded
   d) his favourite numbers 21 and 31 are included and his unlucky numbers 13, 26 and 39 are excluded?

5. How many games are possible if a person chooses:
   a) 3 even numbers and 3 odd numbers
   b) 4 even numbers and 2 odd numbers
   c) 2 even numbers and 4 odd numbers
   d) 4 single digit numbers and 2 double digit numbers
   e) 2 single digit numbers and 4 double digit numbers ending and/or beginning in 3?

6. How many games are possible if a person chooses:
   a) more than 4 even numbers
   b) less than 4 even numbers
   c) at least 1 even number
   d) at least 2 even numbers
   e) at least 3 multiples of 5
   f) at least 3 multiples of 8
   g) at least 4 single digit numbers
   h) at least 4 double digit numbers?

7. Ignoring the order in which the main numbers are drawn and the order in which the supplementary numbers are drawn, in how many different ways can:
   a) the main numbers be even and the supplementary numbers be odd
   b) the main numbers be odd and the supplementary numbers be even
   c) the main numbers are less than 25 and the supplementary numbers are 25 or greater
   d) all 8 numbers be even (NB. 26 as a main number is different from 26 as a supplementary number)?

TEAM / COMMITTEE PROBLEMS

A Mathematics C class consists of:

- males - Angus, Brandon, Brett, James, Mark, Paul, Peter, Tony
- females - Janelle, Kelly, Rowena, Sarah.

8. A team of 5 is selected to represent the School at a Mathematics seminar. How may different teams can be selected if the team contains:
   a) contains all males
   b) contains all females
   c) includes the eldest student
   d) excludes the eldest student
   e) includes the eldest student and excludes the youngest student
   f) includes all students whose name begins with B
   g) excludes all students whose name begins with B
   h) contains all students whose name begins with B and excludes all students whose name begins with P
   i) includes at least one female
   j) contains 1 male and 4 females
   k) contains 1 female and 4 males
   l) includes more males than females
   m) includes more females than males?

9. Three males and three females are to be chosen as the School team in a Mathematics competition and the team is to be arranged in a row. In how many ways can this be done if:
   a) all the males sit together
   b) all the males sit together and all the females sit together ?

10. Repeat (b) in the previous question if 4 males and 2 females are chosen and the team sits around a circular table.

PLAYING CARDS

In the following, hands of cards are dealt from a well shuffled pack of 52 cards.

11. How many different poker hands of 5 cards:
   a) consist of all hearts
   b) consist of cards of the same suit (flush)
   c) contain 4 of a kind eg. 4 kings and another card
   d) contain 3 of one kind and 2 of another kind (full house) eg. 3 kings and 2 fives ?

12. Peter, Tony, Sarah and Rowena are playing bridge, a game in which each person is dealt 13 cards. In how many different ways can the cards be dealt to these four players?

MISCELLANEOUS

13. I have 7 friends and I have bought 7 different Christmas presents. In how many ways can I give the presents if:
    a) each friend receives 1 present
    b) 1 friend receives 2 presents and the other presents are each given to one person
    c) 2 friends each receive 2 presents and the other presents are each given to one person ?

ANSWERS - MULTIPLICATION AND ADDITION PRINCIPLES

(1) (a) 5040 (b) 720 (c) 1440 (d) 240 (e) 3600 (f) 2400 (g) 720 (h) 1440 (i) 720
(2) (a) 13699 (b) 960799
(3) (a) 362880 (b) 60480 (c) 100800 (d) 2880 (e) 100800 (f) 17280 (g) 14400 (h) 2880
(4) (a) 260649 (b) 5380839 (5) (a) 479001600 (b) 21772800 (c) 103680 (d) 43545600 (e) 54432000
(6) (a) 720 (b) 48 (c) 240 (d) 48 (7) (a) 120 (b) 48 (c) 16 (8) (a) 362880 (b) 17280 (c) 1728 (9) (a) 40320 (b) 2880 (c) 576 (10) (a) 4500 (b) 1800 (11) (a) 360 (b) 1296 (12) (a) 120 (b) 432
(13) (a) 720 (b) 2058 (14) (a) 320 (b) 882 (15) (a) 156 (b) 258 (16) (a) 223 (b) 343
(17) (a) 104 (b) 172 (18) (a) 161 (b) 245 (19) 60 (20) 6720 (21) 1956 (22) 8268676

ANSWERS - COMBINATIONS

(1) 8145060 (2) (a) 74613 (b) 3003 (c) 84 (d) 27132 (e) 3003
(3) (a) 1086008 (b) 7059052 (c) 820 (d) 4496388 (4) (a) 123410 (b) 1925196 (c) 5245786 (d) 91390
(5) (a) 2727340 (b) 1850695 (c) 2045505(d) 79380 (e) 25740
(6) (a) 680295 (b) 5614070 (c) 8044113 (d) 7303835 (e) 683760 (f) 102740 (g) 84000 (h) 7461300
(7) (a) 18877089 (b) 23318757 (c) 28265160 (d) 8953560
(8) (a) 56 (b) 0 (c) 330 (d) 462 (e) 210 (f) 120 (g) 252 (h) 56 (i) 736 (j) 8 (k) 280 (l) 672 (m) 120
(9) (a) 32256 (b) 16128 (10) 20160 (11) (a) 1287 (b) 5148 (c) 624 (d) 3744 (12) $5{\cdot}36 \times 10^{28}$
(13) (a) 5040 (b) 105840 (c) 264600