Sheet C

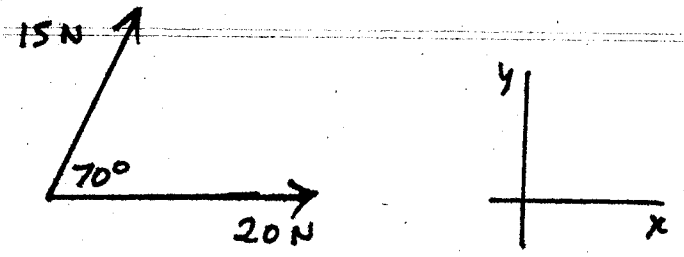
(1)(a) resultant

$$= \begin{pmatrix} 20 \\ 0 \end{pmatrix} + \begin{pmatrix} 15\cos 70° \\ 15\sin 70° \end{pmatrix}$$

$$= \begin{pmatrix} 25.13030215 \\ 14.09538931 \end{pmatrix}$$

$$= (28.8,\ 29.3°)$$

= 28.8 N. making angle 29.3° with the 20 N.



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(2) by Cosine Rule:

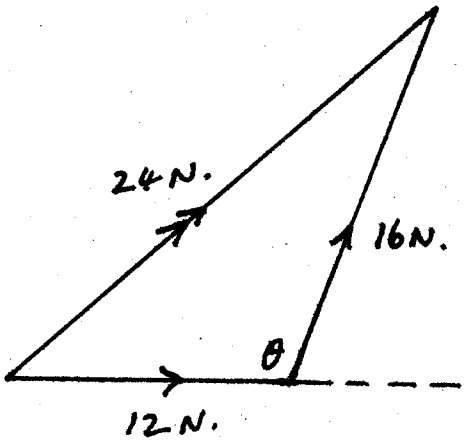
$$\cos\theta = \frac{12^2 + 16^2 - 24^2}{2 \times 12 \times 16}$$

$$\theta = 117.3°$$

∴ angle between forces

= 180° − 117.3°

= 62.7°



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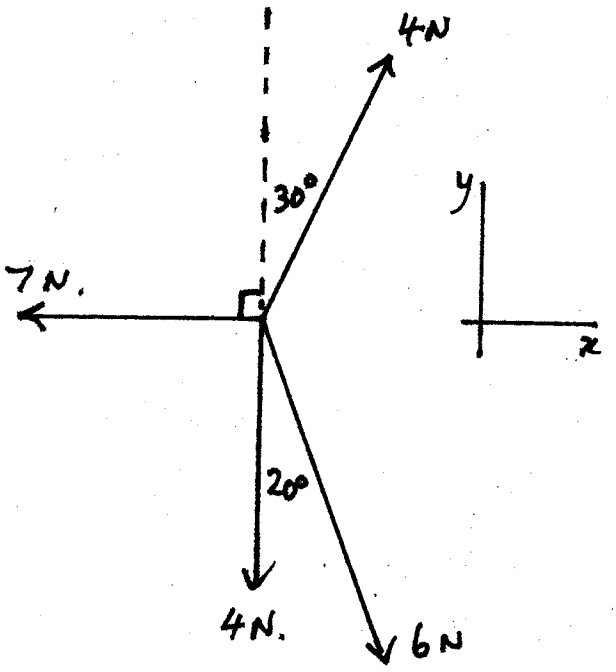
(4)(b) resultant

$$= \begin{pmatrix} 4\cos 60° \\ 4\sin 60° \end{pmatrix} + \begin{pmatrix} 6\cos 290° \\ 6\sin 290° \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} + \begin{pmatrix} -7 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -2.94787914 \\ -6.17405411 \end{pmatrix}$$

$$= (6.84,\ -115.5°)$$

= 6.84 N. on a bearing of 205.5°



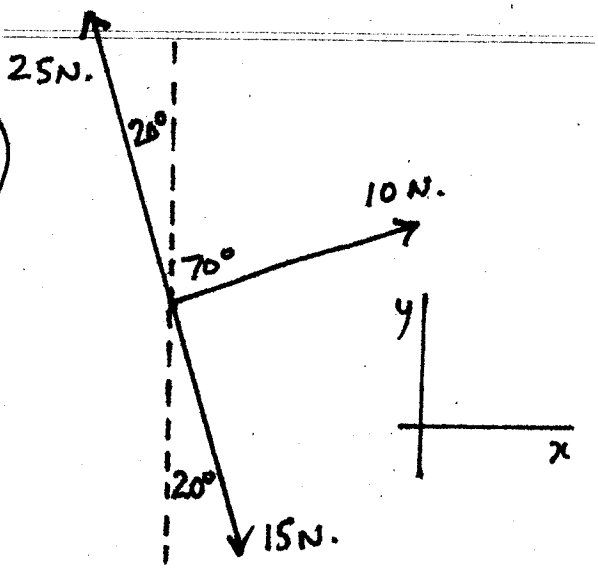
(4)(c) resultant

$$= \begin{pmatrix} 10\cos 20^\circ \\ 10\sin 20^\circ \end{pmatrix} + \begin{pmatrix} 15\cos 290^\circ \\ 15\sin 290^\circ \end{pmatrix} + \begin{pmatrix} 25\cos 110^\circ \\ 25\sin 110^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 5.976724775 \\ 12.81712764 \end{pmatrix}$$

$= (14.1, 65^\circ)$

$=$ 14.1 N. on a bearing of 025°



(5) R(→): $P\sin 30^\circ = 50$

$$P = \frac{50}{\sin 30^\circ}$$

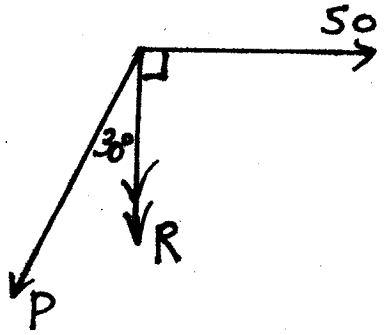
$$P = 100$$

R(↓): $R = P\cos 30^\circ$

$R = 100\cos 30^\circ$

$R = 86.6$

∴ P is 100 and R is 86.6



(7) R(→): $P\sin 10^\circ = 50\sin 60^\circ$

$$P = \frac{50\sin 60^\circ}{\sin 10^\circ}$$

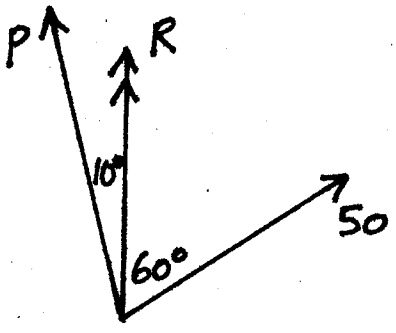
$$P = 249.3620766$$

R(↑): $R = P\cos 10^\circ + 50\cos 60^\circ$

$R = 249.3620766\cos 10^\circ + 50\cos 60^\circ$

$R = 270.5737064$

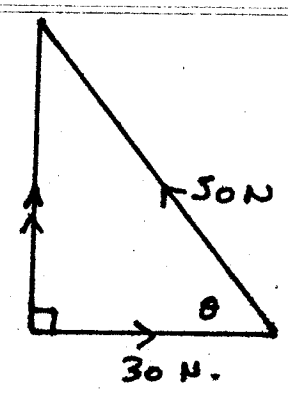
∴ P is 249.4 and R is 270.6



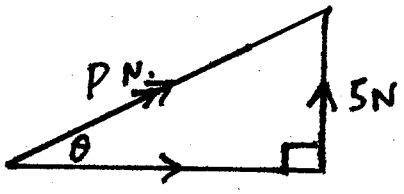
(9) $\cos\theta = \frac{3}{5}$

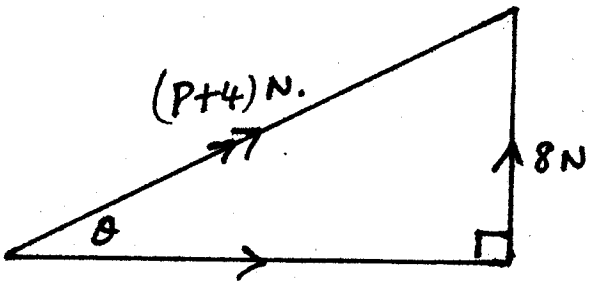
$\theta = 53.1°$

angle between ropes
$= 180° - 53.1°$
$= 126.9°$



(11) Let the original force be P N. making angle θ with the horizontal.





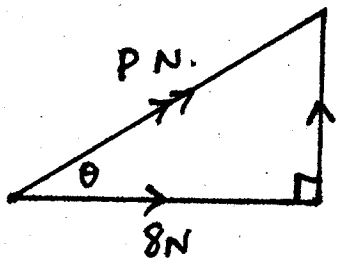
The triangles are similar (equal angles).

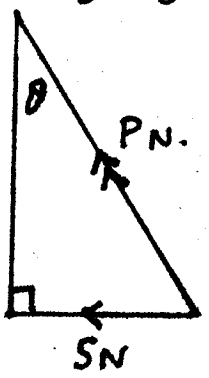
$$\therefore \quad \frac{P+4}{P} = \frac{8}{5}$$

$$5(P+4) = 8P$$
$$5P + 20 = 8P$$
$$20 = 3P$$
$$P = 6\tfrac{2}{3}$$

$\therefore$ Magnitude of force is $6\frac{2}{3}$ N.

(12) Let the force be P N. originally making angle θ with horizontal.





After turning through 90°, the force will make angle θ with the vertical.

The two triangles are congruent. (AAS)

by Pythagoras' Theorem:

$$P^2 = 8^2 + 5^2$$
$$P = 9.43$$

∴ Magnitude of force is 9.43 N.

Sheet D

(1) (a) R(// slope):

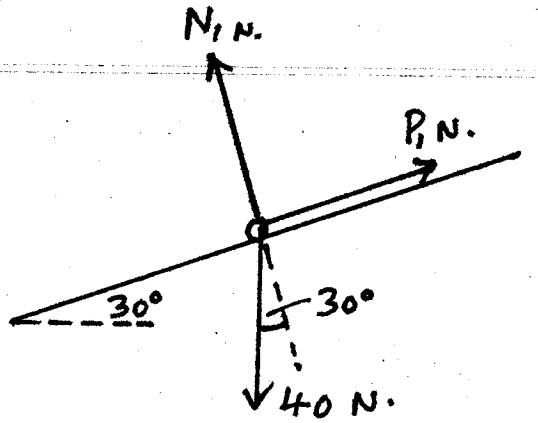
$P_1 = 40\sin 30°$

$P_1 = 20$

R(⊥ slope):

$N_1 = 40\cos 30°$

$N_1 = 34.6$



∴ Force required is 34.6 N and the reaction is 20 N.

(b) R(// slope):

$P_2\cos 30° = 40\sin 30°$

$P_2 = \dfrac{40\sin 30°}{\cos 30°}$

$P_2 = 23.09401077$

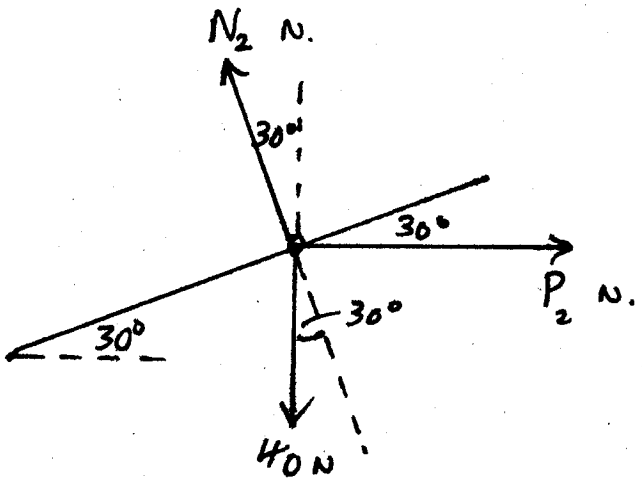
∴

R(↑):

$N_2\cos 30° = 40$

$N_2 = \dfrac{40}{\cos 30°}$

$N_2 = 46.18802154$



∴ Force required is 23.1 N and the reaction is 46.2 N.

---

(3) Let the mass be m kg.

R(// slope):

$10m\sin 40° = 20\cos 30°$

$m = \dfrac{20\cos 30°}{10\sin 40°}$

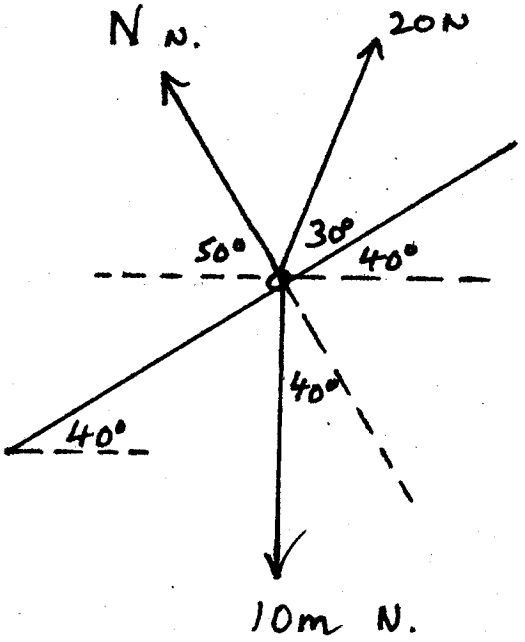
$m = 2.694592711$

R(→):

$N\cos 50° = 20\cos 70°$

$N = \dfrac{20\cos 70°}{\cos 50°}$

$N = 10.64177772$



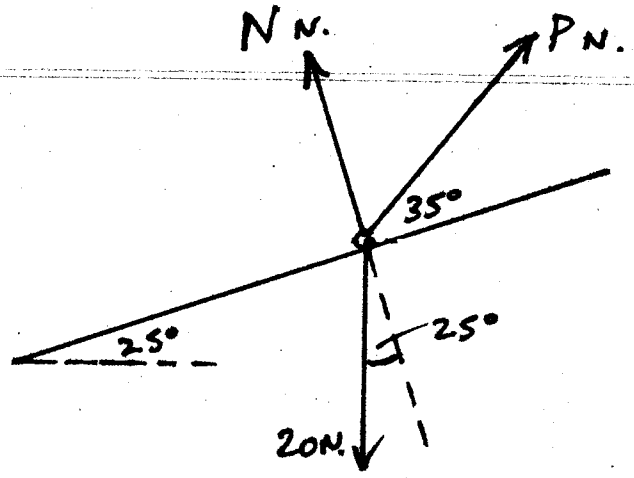
∴ mass is 2.69 kg and the reaction is 10.6 N.

(4) R(// slope):

$P\cos 35° = 20\sin 25°$

$P = \frac{20\sin 25°}{\cos 35°}$

$P = 10.318\,43269$



R(⊥ slope)

$N + P\sin 35° = 20\cos 25°$

$N = 20\cos 25° - 10.318\,43269\sin 35°$

$N = 12.207\,74589$

∴ P is 10.3 and the reaction is 12.2 N.

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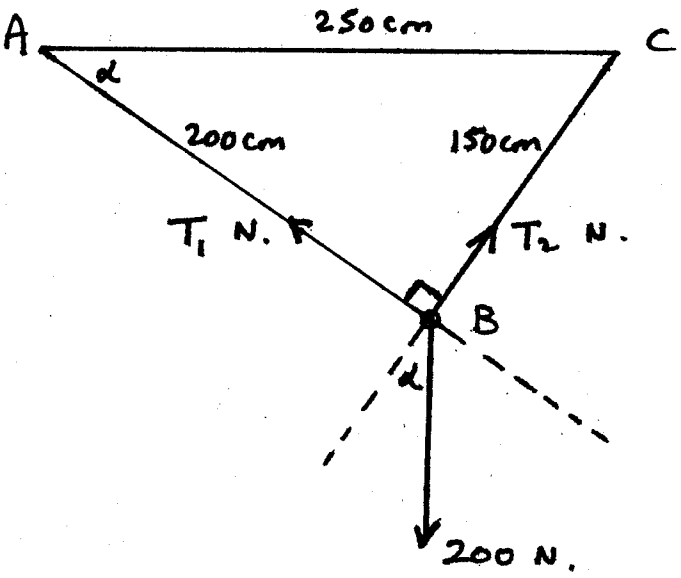
(16) $\angle ABC = 90°$ (3,4,5Δ)

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$



R(⊥ T_2):

$T_1 = 200\sin\alpha$

$T_1 = 200 \times 3/5$

$T_1 = 120$

R(⊥ T_2):

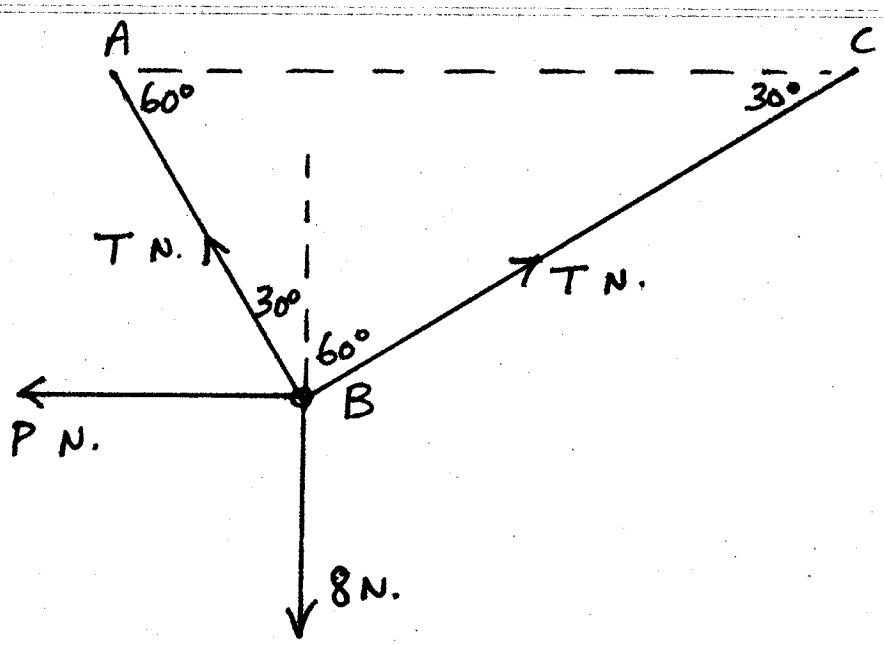
$T_2 = 200\cos\alpha$

$T_2 = 200 \times \frac{4}{5}$

$T_2 = 160$

∴ Tensions are 120 N and 160 N.

(8)



$R(\uparrow)$: $T\cos 30° + T\cos 60° = 8$

$T(\cos 30° + \cos 60°) = 8$

$$T = \frac{8}{\cos 30° + \cos 60°}$$

$T = 5.856406461$

$R(\rightarrow)$: $P + T\sin 30° = T\sin 60°$

$P = T(\sin 60° - \sin 30°)$

$P = 5.856406461(\sin 60° - \sin 30°)$

$P = 2.14359354$

∴ P is 2.14 and the tension is 5.86 N.

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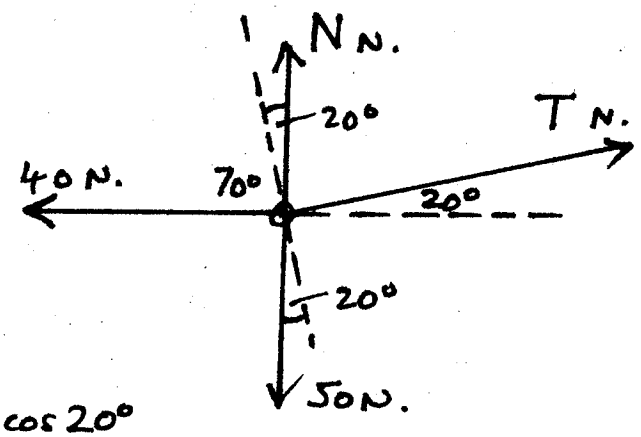
(12) $R(\rightarrow)$:

$T\cos 20° = 40$

$$T = \frac{40}{\cos 20°}$$

$T = 42.5671109$



$R(\perp T)$:

$N\cos 20° + 40\cos 70° = 50\cos 20°$

$$N = \frac{50\cos 20° - 40\cos 70°}{\cos 20°}$$

$N = 35.44119063$

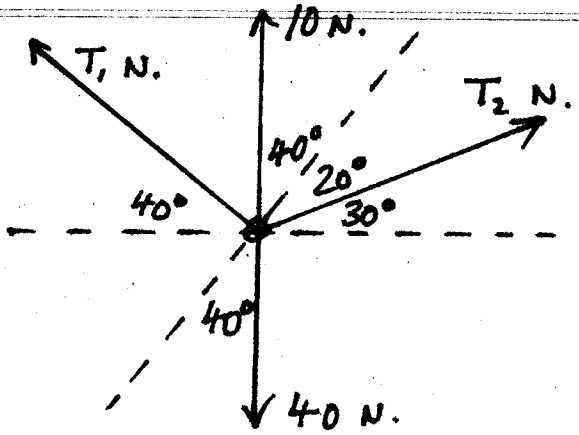
∴ Tension is 42.6 N. and reaction is 35.4 N.

(13) $R(\perp T_1)$:

$T_2\cos 20° + 10\cos 40° = 40\cos 40°$

$T_2 = \dfrac{30\cos 40°}{\cos 20°}$

$T_2 = 24.45622407$



$R(\rightarrow)$:

$T_1\cos 40° = T_2\cos 30°$

$T_1 = \dfrac{24.45622407\cos 30°}{\cos 40°}$

$T_1 = 27.64814955$

∴ Tensions are 24.5 N and 27.6 N.

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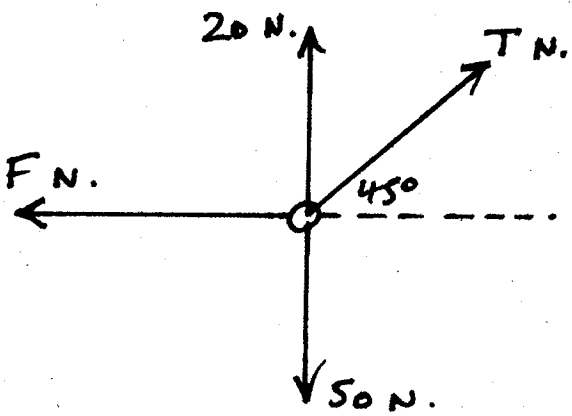
(14) $R(\uparrow)$:

$20 + T\sin 45° = 50$

$T = \dfrac{30}{\sin 45°}$

$T = 42.42640687$



$R(\rightarrow)$:

$F = T\cos 45°$

$F = 42.42640687\cos 45°$

$F = 30$

∴ Tension is 42.4 N and the frictional force is 30 N.