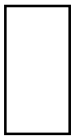
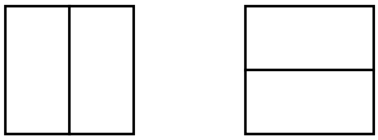
PAVER PROBLEM - SOLUTION 1

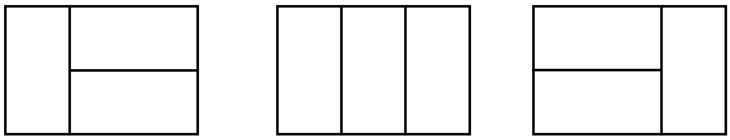
Consider paths of length ½ metre. There is only one possibility:



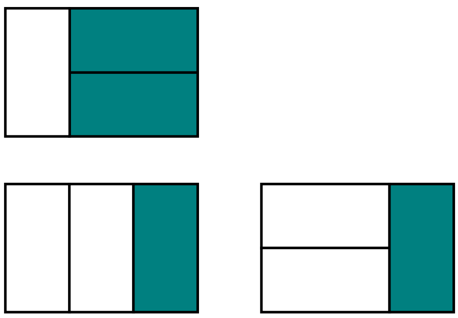
Consider paths of length 1 metre. There are two possibilities:



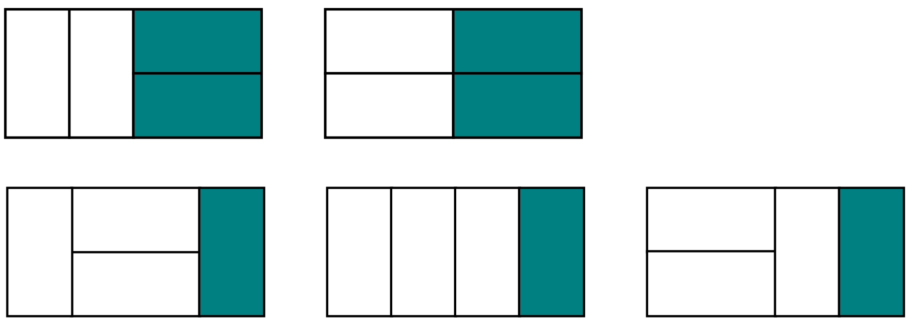
Consider paths of length 1½ metres. There are three possibilities:



All paths of length 1½ metres can be made by adding two pavers to the path of length ½ metre or one paver to paths of length 1 metre:



Similarly, paths of length 2 metres can be made by adding two pavers to paths of length 1 metre or one paver to paths of length 1½ metres:



This method of finding all paths can be continued and results in the numbers of paths shown in the following table:

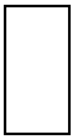
length of path (metres)	number of different paths
½	1
1	2
1½	1 + 2 = 3
2	2 + 3 = 5
2½	3 + 5 = 8
3	5 + 8 = 13
3½	8 + 13 = 21
4	13 + 21 = 34
4½	21 + 34 = 55
5	34 + 55 = 89
5½	55 + 89 = 144
6	89 + 144 = 233
6½	144 + 233 = 377
7	233 + 377 = 610
7½	377 + 610 = 987
8	610 + 987 = 1597
8½	987 + 1597 = 2584
9	1597 + 2584 = 4181
9½	2584 + 4181 = 6765
10	4181 + 6765 = 10946

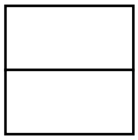
Therefore the number of different ways to lay the pavers for a path of length 10 metres without cutting them is 10946.

The numbers form the Fibonacci sequence $\{f_n\}$ where f_n is the number of different paths of length $\frac{1}{2}n$.

PAVER PROBLEM - SOLUTION 2

The pavers can either be laid as:

a <u>single</u> paver across the path 

or as a <u>pair</u> of pavers along the path  .

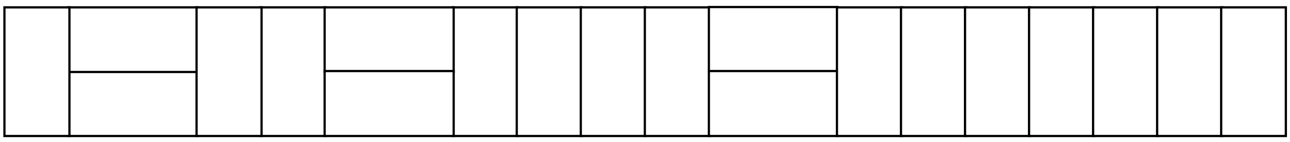
The possible arrangements of singles and pairs are as follows.

number of singles	20	18	16	14	12	10	8	6	4	2	0
number of pairs	0	1	2	3	4	5	6	7	8	9	10

Each arrangement consists of a number of positions taken up by the pairs and singles:

number of singles	20	18	16	14	12	10	8	6	4	2	0
number of pairs	0	1	2	3	4	5	6	7	8	9	10
number of positions	20	19	18	17	16	15	14	13	12	11	10

Consider the arrangement with 17 positions for 14 singles and 3 pairs. One possible path is as follows:



The number of different paths is the number of ways of choosing the 3 positions for the pairs from the 17 available. This number is the combination ${}^{17}C_3$.

Therefore, considering all the possible arrangements:

total number of different paths

$$= {}^{20}C_0 + {}^{19}C_1 + {}^{18}C_2 + {}^{17}C_3 + {}^{16}C_4 + {}^{15}C_5 + {}^{14}C_6 + {}^{13}C_7 + {}^{12}C_8 + {}^{11}C_9 + {}^{10}C_{10}$$
$$= 1 + 19 + 153 + 680 + 1820 + 3003 + 3003 + 1716 + 495 + 55 + 1$$
$$= 10946$$

Therefore the number of different ways to lay the pavers for a path of length 10 metres without cutting them is 10946.