

## HANDSHAKE INVESTIGATION – APPROACH 1

Represent the people as A, B, C, ... and a handshake as AB, BC, ...

Consider 2 people in the room:

AB

There is 1 handshake.

Consider 3 people in the room:

AB BC

AC

There are 3 handshakes.

Consider 4 people in the room:

AB BC CD

AC BD

AD

There are 6 handshakes.

Consider 5 people in the room:

AB BC CD DE

AC BD CE

AD BE

AE

There are 10 handshakes.

These results can be shown in a table:

number of people	number of handshakes
2	1
3	3
4	6
5	10

The numbers of handshakes are the triangle numbers 1, 3, 6, 10:

number of people	number of handshakes
2	1
3	$2 + 1 = 3$
4	$3 + 2 + 1 = 6$
5	$4 + 3 + 2 + 1 = 10$

To explain why the triangle number pattern is appropriate, consider 6 people in the room:

The first person A must shake hands with 5 people:

AB  
AC  
AD  
AE  
AF

B has already shaken hands with A and so only needs to shake hands with 4 more people:

BC  
BD  
BE  
BF

C has already shaken hands with A and B and so only needs to shake hands with 3 more people:

CD  
CE  
CF

D has already shaken hands with A , B and C and so only needs to shake hands with 2 more people:

DE  
DF

E has already shaken hands with A, B, C and D and so only needs to shake hands with 1 more person:

EF

Therefore, when 6 people are in the room, the number of handshakes is the 5th triangle number  $15 = 5 + 4 + 3 + 2 + 1$ .

AB BC CD DE EF  
AC BD CE DF  
AD BE CF  
AE BF  
AF

## HANDSHAKE INVESTIGATION – APPROACH 2

The number of handshakes can also be calculated as shown in the following table.

number of people	number of handshakes
2	$\frac{2 \times 1}{2} = 1$
3	$\frac{3 \times 2}{2} = 3$
4	$\frac{4 \times 3}{2} = 6$
5	$\frac{5 \times 4}{2} = 10$
6	$\frac{6 \times 5}{2} = 15$

This pattern is as follows:

$$\text{number of handshakes} = \frac{(\text{number of people}) \times (\text{number of people} - 1)}{2}$$

The formula for the number of handshakes is justified below.

Each person shakes hands with every other person ie. the number of handshakes for each person is equal to one less than the number of people.

To find the total number of handshakes, multiply the number of people by the number of handshakes for each person but divide the answer by 2 to avoid counting each handshake twice.